A Computationally Efficient Node-Selection Scheme for Cooperative Beamforming in Cognitive Radio Networks

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Abstract—Transmit beamforming is an effective mechanism to enable the operation of secondary users (SUs) in underlay cognitive radio networks. However, wireless nodes that are equipped with single omni-directional antennas cannot exploit the benefits of transmit beamforming on their own. Using cooperative beamforming, a group of nodes can collaborate to steer the signal towards the direction of the intended receiver while, at the same time, null the transmitted signal in the direction of primary receiver. In this paper, we study how to select a subset of $N$ nodes out of $N_T$ nodes for performing cooperative beamforming. Specifically, we investigate how different node-selection schemes affect the performance of cooperative beamforming. Motivated by the findings from this study, we propose a computationally efficient node-selection scheme for achieving a near-optimal performance in cooperative beamforming. Using simulation results, we demonstrate the performance of our proposed scheme, and show that it is scalable with the system size (i.e., for different values of $N$ and $N_T$) and is computationally very efficient as compared to other node-selection schemes.

I. INTRODUCTION

In the last few decades, the growth of wireless communication systems that require large bandwidths has resulted in an increasing demand for radio spectrum. Radio spectrum, being a limited resource, has become increasingly crowded, and there is a need to look for techniques to improve the spectral efficiency. Cognitive Radios are devices that dynamically adapt their system parameters to the surrounding radio environment. In a Cognitive Radio Network (CRN), there are two approaches by which secondary (or unlicensed) users can coexist with the primary (or licensed) users in the same wireless spectrum. Secondary users (SUs) can opportunistically access the radio spectrum when primary users (PUs) are not using it—this is referred to as the overlay system. SUs could also concurrently use the spectrum, if they can ensure that the interference caused to the PU is below a predefined threshold—this is referred to as the underlay system.

With the increase in capabilities of nodes in a CRN, the opportunity to perform complex tasks, such as beamforming, also increases. Transmit beamforming is one of the ways for SUs to access the spectrum in an underlay CRN. The main requirement for performing beamforming is to have multiple antennas at the transmitter device. However, in certain scenarios, such as nodes in a wireless sensor network and other Internet-of-Things (IoT) applications, a transmitter may only be equipped with a single omni-directional antenna, and hence, the transmitter cannot implement beamforming on its own. In such cases, cooperative beamforming [1]- [4] can be used. It allows multiple single-antenna-transmitters to collaborate and steer a message signal towards the desired receiver while limiting interference in the direction of co-channel PUs below a predefined threshold.

An important consideration in wireless sensor networks and IoT applications is conservation of energy. Typically, wireless nodes in sensor networks or IoT applications are installed at distant locations where power supply may be limited, and batteries are used to power the nodes. In such cases, replacing the batteries manually may not be feasible. In order to conserve energy, we may require a subset of the total nodes to participate in beamforming applications. In doing so, however, we wish to maximize the gain received at the secondary receiver (SU-Rx) as well as ascertain that no interference is caused to the PU. In the past, several works have considered performing cooperative beamforming with stationary as well as mobile secondary users [5]–[11]. However, existing works focus mainly on steering the signal towards the SU-Rx using a given set of nodes. None of these works focus on how to select a subset of total available nodes.

In this paper, we study several strategies to select a subset of nodes for performing cooperative beamforming. We first investigate how different node-selection schemes affect the performance of cooperative beamforming. Our analysis shows that the optimum node-selection scheme—i.e., the scheme that achieves the highest Signal to Noise Ratio (SNR) gain towards the SU-Rx—is computationally very expensive, and such a scheme is not scalable to large wireless networks. Motivated by these findings, we propose a computationally efficient node-selection scheme for achieving a near-optimal performance in cooperative beamforming. The main contributions of our work are as follows:

- We study how different node-selection schemes affect the performance of cooperative beamforming in network. Our findings show that the optimum node-selection scheme—i.e., the scheme that achieves the highest Signal to Noise Ratio (SNR) gain towards the SU-Rx—is computationally very expensive, and such a scheme is not feasible to implement in large wireless networks.
- We propose a computationally efficient node-selection scheme for achieving a near-optimal performance in
cooperative beamforming. The proposed scheme is highly scalable to the size of wireless networks. We demonstrate the efficacy of our proposed scheme using detailed simulation results.

II. RELATED WORK

Previous work on transmit beamforming mostly considered the case where wireless nodes are equipped with multiple antennas. Assuming that the complete channel state information (CSI) is available at the receiver, researchers studied transmit beamforming for a stationary SU-Rx in [5]–[10]. Reference [11] focuses on transmit beamforming for a moving SU-Rx. Specifically, the authors propose a scheme for maintaining a constant transmission power across the range of SU-Rx’s mobility. Other works on transmit beamforming study approaches that are robust against uncertainties in steering vector and CSI information.

Given a set of cooperative nodes, each equipped with a single omni-directional antenna, the authors of [12] propose a scheme for computing the optimum beamforming weights. The proposed scheme of [12] works for both stationary as well as a moving SU-Rx, and it is robust against uncertainties in steering vector and CSI. While existing work focuses on cooperative transmit beamforming, there is no existing work that discusses the effect of node-selection schemes on performance of cooperative transmit beamforming. To the best of our knowledge, this is the first work that proposes a computationally efficient scheme for choosing an optimal set of nodes for performing cooperative beamforming in a large wireless network.

III. SYSTEM MODEL AND ASSUMPTIONS

We consider an underlay cognitive radio network where SUs coexist with co-channel PUs without causing interference to the latter. Specifically, as shown in Figure 1, we consider a secondary transmitter trying to communicate with a stationary secondary receiver.

We make the following assumptions,

- The PU and SU operate in the same frequency band.
- Each SU has only one omnidirectional antenna.
- All SUs agree to cooperate with each other for generating beamforming.
- SUs are uniformly distributed in a circle of radius $R$.
- The CSI is known at each secondary cooperating device.
- All SUs operate at the same transmission power.
- Location information of all PUs and SUs is shared among each other.
- We assume that the PU is stationary.
- We consider simplified path loss model with shadowing to model the propagation environment between the cooperating SUs and the secondary and primary receivers.
- We make no assumptions about the particular technology used by primary and secondary devices.

In order to analyze the performance of the SU-Rx, we consider the received SNR as the performance metric. Keeping in mind the above scenario and assumptions, our objective is to maximize the SNR at SU-Rx while ensuring that the interference caused to the PU does not exceed a predefined threshold. Figure 2 shows an overview of the desired system. The cooperating nodes steer the signal towards the SU-Rx, while creating a null at the PU.

Given a set of nodes (a subset of all the nodes in the network), we first formulate the cooperative transmit beamforming problem as an optimization problem. Let the total number of nodes in the network be $N_T$, and the number of nodes that can collaborate to achieve cooperative beamforming be $N$. Although we consider only one PU, our problem formulation can be easily extended to cases where there are
multiple PUs. We consider a polar coordinate system, with the center of the circle as the origin. The position of the $n$th secondary node is represented as $(d_n, \phi_n)$, where $n = 1, 2, \cdots, N_T$, $d_n$ and $\phi_n$ are the distance and angle of the node from the center of the circle. The positions of PU-Rx and SU-Rx are represented as $(d_{SU}, \phi_{SU})$ and $(d_{PU}, \phi_{PU})$ respectively. The distance of the $n$th node from the primary and secondary user are denoted by $p_n$ and $s_n$ respectively.

We assume simplified propagation model with exponential path loss and shadowing. Beyond a reference distance $d_0$, the path loss in dB ($P_L$) between two points separated by a distance $d$ is given by,

$$P_L = a + b \log_{10}d + \psi,$$  \tag{1}

where $a = P_L(d_0) - b \log_{10}d_0$, $P_L(d_0)$ is the path loss at the reference distance in dB, $b = 10\gamma$, where $\gamma$ is the path loss exponent, and $\psi$ is the log-normal shadowing coefficient with mean 0 and variance $\sigma^2$.

IV. OPTIMAL NODE-SELECTION SCHEME FOR COOPERATIVE BEAMFORMING

A. Cooperative Beamforming

The steering vector of the cooperating nodes towards an arbitrary point is defined as the vector consisting of relative delays of the signal originating from the point, and received at each of the cooperating nodes. For a given set of cooperating nodes, the steering vector towards a point $(d, \phi)$ is given as,

$$\alpha(d, \phi) = [\exp(-j\omega_1/c), \cdots, \exp(-j\omega_N/c)]$$  \tag{2}

where $\omega = 2\pi f_c$, $f_c$ is the carrier frequency in Hz, and $a_i$ is the distance of the $i$th cooperating node, $i = 1, 2, \cdots, N_T$ from the point $(d, \phi)$. For example, the steering vector towards SU-Rx is

$$\alpha(d_{SU}, \phi_{SU}) = [\exp(-j\omega s_1/c), \cdots, \exp(-j\omega s_N/c)].$$

We denote $\alpha_s = \alpha(d_{SU}, \phi_{SU})$ and $\alpha_p = \alpha(d_{PU}, \phi_{PU})$ as the steering vector towards SU-Rx and PU-Rx respectively.

The distance $a_n$ can be computed as follows,

$$a_n = \sqrt{d^2 + d_n^2 - 2dd_n \cos (\phi - \phi_n)}.$$  \tag{3}

Assuming $d \gg d_n$, $a_n$ can be approximated as,

$$a_n \approx d - d_n \cos (\phi - \phi_n).$$  \tag{4}

The channels from all cooperating nodes are assumed to be Rayleigh fading channels. Let the Rayleigh fading channel coefficient for an $i$th node, $i = 1, 2, \cdots, N$, towards SU-Rx and PU for the be be $h_s$, and $h_p$ respectively. Let $h_s = [h_{s1}, h_{s2}, \cdots, h_{sN}]$, and $h_p = [h_{p1}, h_{p2}, \cdots, h_{pN}]$.

If the beamforming weight for the $n$th node is $w_i$, and $\mathbf{w} = [w_1, w_2, \cdots, w_N]$, then the received signal at SU-Rx is given by,

$$y_s = H_{ss} \mathbf{w}x + \psi_1$$  \tag{5}

where $H_{ss} = \text{diag}(h_s)\alpha_s$, where $\text{diag}(h_s)$ is a diagonal matrix of size $N \times N$ with elements of $h_s$ as the diagonal elements, and $\psi_1$ represents additive white Gaussian noise with zero mean and variance $\sigma^2$.

Similarly, the interference signal received by PU-Rx, $y_p$, from cooperative transmitter cluster is,

$$y_p = H_{pu} \mathbf{w}x + \psi_2$$  \tag{6}

where $H_{pu} = \text{diag}(h_p)\alpha_p$, where $\text{diag}(h_p)$ is a diagonal matrix of size $N \times N$ with elements of $h_p$ as the diagonal elements, and $\psi_2$ represents additive white Gaussian noise with zero mean and variance $\sigma^2$.

Let the maximum interference tolerable at the PU-Rx be $I_P$. This implies that while the cooperating secondary transmitters steer the signal towards SU-Rx, the total signal received at PU-Rx should be less than $I_P$.

The overall beamforming problem is formulated as an optimization problem, where we compute the beamforming weights $\mathbf{w}$, such that

Maximize $||H_s \mathbf{w}||$,

subject to $||H_p \mathbf{w}|| \leq I_P$,

$$||w_i||^2 = 1.$$

The first constraint implies that the total signal power received from all cooperating secondary nodes is less than the maximum permissible interference with which the PU-Rx can successfully decode the signal. The second constraint on the weight vector implies that the total transmission power of each cooperating secondary node is the same, and the beamforming weights only alter the phase of the transmitted signal.

In order to convert the optimization problem (7) to a convex optimization problem, we modify the objective function and add a new a constraint such that the overall optimization problem becomes,

Maximize $\Re \{H_s \mathbf{w}\}$,

subject to $||H_s \mathbf{w}|| \leq I_P$,

$$||w_i||^2 = 1, \quad \Re \{H_s \mathbf{w}\} = 0.$$  \tag{8}

For a given set of cooperating secondary nodes, the optimization problem (8) is solved using the interior point algorithm implemented in Matlab Optimization Toolbox to obtain the beamforming weights.

B. Node-Selection Schemes

In the previous subsection, we discussed – given a set of nodes, how to formulate the optimization problem in order to perform cooperative beamforming to steer the signal towards SU-Rx and null the signal at PU-Rx. However, this set of nodes may not provide the maximum possible objective function. We now discuss a few approaches to select the subset of cooperating nodes such that while satisfying all constraints, the gain at SU-Rx is the highest possible.

1) Random Selection of $N$ Nodes: Since the total number of nodes in the network are $N_T$, and only $N$ of these nodes can participate in cooperative beamforming, there are a total of $\binom{N_T}{N} = \frac{N_T!}{N!(N_T-N)!}$ combinations of nodes that can be used. The optimization problem (8) can be solved for most of these
combinations. The simplest approach to select the nodes would be to randomly choose one among the \( \binom{N_T}{5} \) combinations, and compute the performance. If the optimization problem (8) does not converge, we choose another set of nodes randomly until the optimization problem gives a valid solution. However, while all the constraints will be satisfied, the value of the objective function (i.e. gain at the SU-Rx) may not be sufficiently large.

2) Exhaustive Search Among All Node Combinations: In order to obtain the ideal set of nodes that provide the largest gain at the SU-Rx, the only mechanism is to perform an exhaustive search of all node combinations, and run the optimization problem (8) for each node combination. This will require solving the optimization problem \( \binom{N_T}{5} \) times. While this may be feasible for small system sizes, the number of computations become increasingly high with increasing system sizes. For example, with \( N_T = 10 \) and \( N = 5 \), we need to solve the optimization problem 252 times. On the other hand, with \( N_T = 20 \) and \( N = 5 \), we need to solve the optimization problem 15504 times. It must be noted that a set of nodes may be optimal only for a particular set of channel state at the cooperating secondary transmitters. If the channel at any of the transmitters changes, the nodes may not be optimal any longer. Thus, performing such large number of computations in real-time is not feasible, and there is a need for approaches to search for set of nodes that provide near optimal gain at SU-Rx with lesser computational complexity.

3) Selection of Nodes With Highest Channel State Towards SU-Rx: One approach to select \( N \) nodes from \( N_T \) nodes would be to select the \( N \) nodes that have the maximum value of channel state in the direction of secondary receiver, i.e. select the nodes with maximum value of \( |H_{su}| \). However, while this set of nodes may be the ideal candidate for cooperative beamforming in the absence of any primary receiver, satisfying the constraints of (8) may produce beamforming weights such that the gain towards SU-Rx may not be optimal.

4) Proposed Scheme: The gain at SU-Rx primarily depends on the choice of participating nodes. A certain node may perform poorly with an arbitrary set of \( N - 1 \) nodes, but it may perform extremely well with some other set of \( N - 1 \) nodes that are selected from the remaining \( N_T - N \) nodes. Thus, if a given node performs poorly, we cannot declare that the node cannot exist in the set of optimal nodes. This forms the basis of our proposed algorithm for choosing near optimal set of nodes, i.e. nodes that approximately achieve similar performance as that by the optimal set of nodes, but with a much lower computational complexity.

Now consider, for example, there are 12 nodes available in the network out of which we have to pick 5 nodes to achieve cooperative beamforming. The total combinations possible are \( \binom{12}{5} = 792 \). In order to achieve high computational efficiency, we need to sacrifice on the gain in the direction of SU-Rx. Now, suppose out of these 792 combinations, 20 combinations give us close to optimal performance (say within 1 dB of the optimal gain). If we pick 5 nodes at random, the probability of picking a bad set of nodes (say, a set of nodes that gives a gain of 1 dB or more lower than the optimal set) is \( \frac{772}{792} = 0.9747 \), which is very high. If, however, we pick a set of 5 nodes independently 20 times, then the probability of selecting a bad set of nodes in each of these 20 iterations is \( \left( \frac{772}{792} \right)^{20} = 0.2783 \), which is substantially lower.

We consolidate the above two arguments— (i) a particular node may provide very high gain with a certain set of nodes, but may perform poorly with certain other set of nodes, and (ii) when multiple independent set of \( N \) nodes are used to achieve cooperative beamforming, the probability of all the sets performing poorly is substantially lower than a randomly selected set of node giving poor performance. We use these insights into our proposed algorithm. We first select a set of \( N \) nodes, and compute the beamforming weights as described in Section IV-A. Next, we determine the node that contributes the minimum towards achieving cooperative beamforming. This node is replaced by a node from the remaining \( N_T - N \) nodes, and the resulting set is used to achieve cooperative beamforming. We again determine the node that contributes the minimum towards achieving cooperative beamforming, and replace this node from the remaining \( N_T - N \) nodes. The node that was earlier discarded may be picked now, and could perform well with the new set. Thus, we repeat this procedure for \( iterations\_threshold \) number of times, and pick the set of nodes that performs the best in these iterations. To improve the performance further, we ensure that a particular set of nodes is not repeated across iterations.

Our proposed algorithm is summarized as in Algorithm 1.

Algorithm 1 Proposed Node-Selection Algorithm

1: Select threshold variable \( iterations\_threshold \)
2: for \( i \) in \( \{1 \cdots iterations\_threshold\} \) do
3: Choose a random set of nodes \( N \) from \( N_T \) nodes.
4: Solve the optimization problem (8) for this set of nodes.
5: Find the node from this set that contributes the minimum towards the objective function (i.e. \( |H_{su,w}| \)).
6: Replace this node with a node chosen randomly from the remaining \( N_T - N \) nodes.
7: Sort the nodes in ascending order of index.
8: if Node combination has been selected before, then
9: Go back to step 6.
10: end if
11: end for
12: Return the set of nodes that perform the best among all other \( iterations\_threshold \) sets.

The performance, as well as the computational complexity, of our proposed algorithm depends on the choice of the threshold variable \( iterations\_threshold \). It is agnostic of the network size, \( N_T \) and \( N \), which makes this approach lucrative to use in large networks. We justify this argument by providing simulation results in the next section.
V. SIMULATION RESULTS

In this section, we provide detailed simulation results to demonstrate the performance of the proposed node-selection scheme for cooperative beamforming. Let us assume that the cooperative nodes are distributed uniformly in a circular area of radius, $R = 75$ meters. It is assumed that each node is equipped with a single omni-directional antenna. Throughout the simulations, we assume that the intended SU-Rx located in a direction of $30^\circ$ from the reference direction and at a distance, $d_{su} = 1$ km. Also, we assume that a single PU-Rx is located in a direction of $80^\circ$ from the reference direction at a distance, $d_{pu} = 1$ km.

We consider the following RF parameters in our simulations. Let $f_c = 900$ MHz, $\gamma = 2.5$ and $\sigma = 3$ dB. Let the interference tolerance threshold of the PU-Rx, $I_p = -80$ dBm. Each participating node in cooperative beamforming transmits with a transmit power of $P_{tx} = 23$ dBm. Noise level is set to $-105$ dBm. We assume that each node experiences an uncorrelated Rayleigh fading channel in each direction. We consider $360^\circ$ directions for each node and generate the channel independently for each direction.

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A. Performance of Cooperative Beamforming

In order to analyze the performance of cooperative beamforming, we generated $N = 5$ nodes that are randomly located in a circle of radius $R = 75$ m, and solved optimization problem (8). We repeated this for 500 iterations, computed the average beamforming gain in different directions and plotted the results. Figure 3 summarizes our observation. Clearly, the beamforming gain towards the SU-Rx is maximized, and a null is generated in the direction of the PU-Rx. Note that we do not observe symmetry in the beamforming gain unlike that in case of a phased array. This is because of our assumption that the participating nodes are not equally spaced; their locations are entirely random within a defined circular area. The sharp peak and the null towards SU-Rx and PU-Rx respectively are consequences of the uncorrelated channel assumption (recall that, for each node, we assumed independent channel in different directions). Furthermore, Figure 3 verifies that the cooperative beamforming considered in this study achieves the dual objectives of a cognitive radio network—(i) maximize the signal at the intended receiver, and (ii) minimize interference at the PU-Rx.

B. Analyzing the Performance of the Proposed Scheme

Here, we analyze the performance of the proposed scheme and compare it against all other schemes discussed in Section IV-B. Specifically, we compare the cumulative distribution function (CDF) of SNR at SU-Rx and the CDF of received signal strength (RSS) at PU-Rx for different node-selection schemes. Specifically, we set $N_T = 15$, and iterations_threshold variable of the proposed scheme is set to 10. The results are summarized in Figures 4 and 5.

From Figure 4, we can be notice that the performance of random node-selection scheme is the worst while the performance of the exhaustive search is the best among all schemes. The scheme that chooses nodes with maximum $|H_{su}|$ performs better than random node-selection scheme, but it is not as good as the proposed scheme. Interestingly, the performance of the proposed scheme is comparable to that of exhaustive search. Note that, for this simulation set up, exhaustive search requires 3,003 iterations while the proposed scheme achieves the reported performance in only 10 iterations (iterations_threshold = 10). Also, from Figure 5, we observe that the proposed scheme, as well as others, satisfies the protection requirement of the PU-Rx—i.e., the proposed scheme is able to constrain the interference power at PU-Rx below the predefined interference tolerance threshold, $I_p = -80$ dBm. In summary, our results validate the efficacy of the proposed node-selection scheme in achieving a near-optimal solution for node-selection in cooperative beamforming. The significantly low computational complexity is the most attractive feature of the proposed scheme.

C. Asymptotic Performance of the Proposed Scheme

In this section, we use simulation results to analyze the asymptotic performance of the proposed node-selection scheme. For performing this study, we set $N_T$ to 15. Figures 6 and 7 summarize our findings. From Figure 6, we observe that, as we increase the number of iterations (iterations_threshold) in the proposed scheme, the distribution of performance of the proposed scheme closely resembles that of exhaustive search. Intuitively, for
iterations_threshold = 1, the proposed scheme is equivalent to the random node-selection scheme. However, as we increase iterations_threshold, the CDF curve shifts rapidly to the right. This provides us an insight about the relation between iterations_threshold and the performance of the proposed scheme—with an increase in iterations_threshold, the proposed scheme can perform as good as the exhaustive search. Figure 7 validates this claim. Clearly, the asymptotic performance of the proposed scheme quickly approaches the limit (the performance of the exhaustive search), and it follows the law of diminishing returns. Table I shows the reduction in computational complexity achieved through our proposed algorithm (in terms of number of iterations) as opposed to exhaustive search, and the resulting performance.

VI. CONCLUSIONS

In this paper, a computationally efficient node-selection scheme is proposed for cooperative transmit beamforming in a cognitive radio network. The proposed scheme provides a near-optimal beamforming solution while significantly reducing the computational burden. Specifically, a reduction in complexity from $\binom{N_T}{N}$ to iterations_threshold is achieved, where iterations_threshold << $\binom{N_T}{N}$. Our simulation results show that the performance achieved by the proposed scheme is approximately 95% of the optimal solution. Our proposed solution is effective in choosing an optimal set of nodes for cooperative beamforming in a large wireless networks.

REFERENCES


| TABLE I

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<th>Node-Selection Scheme</th>
<th>Iterations</th>
<th>Average SNR at SU-Rx (dB)</th>
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